An Outline of Possible In-course Diagnostics for Elementary Logic, Limits and Continuity of a Function

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ABSTRACT This paper focuses on in-course sample diagnostic questions relating to elementary logic, and the two concepts of limits and continuity of a function. These are for students who choose to take a course on differential calculus, in a South African context. However, the diagnostic questions could be useful worldwide. Learning outcomes and in-course diagnostic questions for the technical knowledge and skills required for the indicated sections were formulated. The questions were designed to check for the relevant learning outcomes that the researcher detected for the indicated sections as informed by the literature review and conceptual framework. The learning outcomes and formulation of diagnostic outcomes, although based on a number of assumptions, should improve the performance of students. Investigations into the validity of those assumptions, including their attainment levels and student performance correlations to relevant examination questions will be the focus of another paper.

INTRODUCTION

At the University of KwaZulu-Natal (UKZN) the relatively low pass rates for the first year differential calculus (Math130) module was of concern. The paper by Maharaj and Wagh (2014) focused on the pre-course diagnostics for differential calculus. In this paper the focus is on outlining possible in-course diagnostics for the sections on elementary logic and limits and continuity of functions for differential calculus. These are aimed at assessing the strengths and shortcomings of technical knowledge and skills, of the student. The diagnostic tests would not be for grading students but rather to provide feedback on their strengths and weaknesses with regard to content and skills on elementary logic, limit of a function and continuity of a function. Detailed learning outcomes for the sections on elementary logic, limit of a function and continuity of a function for the first year differential calculus course offered at the UKZN are provided.

The research questions that were focused on are: *What are the expected learning outcomes with regard to the sections on elementary logic and the two concepts, limit and continuity of a function? How could in-course diagnostics on elementary logic, and the limit and continuity of a function for differential calculus be developed?*

This paper will be of interest to readers outside South Africa for the following reasons: (1) diagnostic testing were and are still widely used at universities in the United Kingdom (Learning and Technology Support Network Maths Team Project 2003) and the State of California in the United States of America (The California State University 2012) to gauge the readiness of students to study calculus; (2) in this paper the focus is on in-course diagnostics for some sections relevant to calculus; (3) the formulation of the sample diagnostic tests presented was informed by international literature on diagnostic testing, elementary logic and the concepts on limits and continuity of a function; (4) these sections are relevant to the study of any course on differential calculus taken by students throughout the world. Further the possible diagnostic tests suggested in this paper could be administered as paper based or computer based, depending on the availability of resources and at least one of these could be regarded as emerging economy resources.

Review of Literature

This focuses on: diagnostic testing; elementary logic; the limit and continuity of a function.

Diagnostic Testing

In other papers, *An outline of possible pre-course diagnostics for differential calculus* (Maharaj and Wagh 2014) and *An outline of possible in-course diagnostics for functions*

(Maharaj pre-print) the researchers discuss, in detail, the rationale for diagnostic testing and how one should go about formulating diagnostic questions. The main points are stated here for the reader: (a) There should be clear learning outcomes for sections and these should be made public (Council of Regional Accrediting Commissions 2004). (b) The expected learning outcomes should guide the formulation of the diagnostic questions (Adam 2006). (c) Many institutions in the United Kingdom made use of pre-course paper based or computer based diagnostic testing (Learning and Technology Support Network Maths Team Project 2003). (d) Diagnostic testing has led to improvement of student performance (Betts et al. 2011). (e) Feedback on diagnostic tests relating to the strengths and weaknesses of a student could help him or her plan and take remedial measures to attend to identified weaknesses (The California State University 2012). It was observed that many institutions used diagnostic testing to gauge the readiness of students to study calculus. In this paper the focus was on the formulation of expected learning outcomes and diagnostic questions for in-course diagnostics relating to elementary logic, and limits and continuity of functions. It is the researcher's opinion that these outcomes and diagnostics could improve the performance of students studying those sections, in particular at the University of KwaZulu-Natal and generally in developing countries.

Elementary Logic

Logic is concerned with the principles of correct reasoning (Lau and Chan 2013a). So, elementary logic is concerned with the *basic principles of correct reasoning*. Elementary logic in the context of mathematics should include the principles governing the validity of arguments. In particular the focus should be on whether certain conclusions follow from some given assumptions. In mathematics *formal logic* is used. This is mainly concerned with (a) specially constructed systems for carrying out proofs, and (b) the languages and rules of reasoning which are precisely and carefully defined (Lau and Chan 2013a). It could be argued that logic helps us identify patterns of good reasoning and patterns of bad reasoning. The former is the one which a student should follow, while the latter should be avoided. It is reasonable to assume that study-

ing basic logic can help improve critical thinking. It was found that at UKZN some colleagues believe that elementary logic should be taught as a separate section. Others are of the view that the principles of correct reasoning should be taught in the context of mathematical proofs and the correct writing out of solutions to problems. The crucial point is that in constructing an argument it is important to know how one statement is related to another (Lau and Chan 2013b). This brings us to the concept of a good argument. A good argument has the following characteristics: (a) it is either valid or strong, (b) has plausible premises that are true (in mathematics the assumptions or 'if' condition(s)), (c) the premises must not beg the question (there should be no circular reasoning), and (d) the premises must be relevant to the conclusion (Lau and Chan 2013c).

Limit and Continuity of a Function

An earlier paper (Maharaj, 2010) which dealt with *students' understanding of the concept of a limit of a function* noted that: (a) many past studies focused on the students' understanding of the concept of a limit of a function (for example Cornu 1991; Davis and Vinner 1986; Li and Tall 1993; Maharajh et al. 2008; Monaghan et al. 1994; Tall 1992; Tall and Vinner 1981; Williams 1991); (b) students have difficulties with the concept of a limit of a function in the context of functions and continuity; (c) many of the difficulties encountered by students in dealing with other calculus concepts; for example continuity, differentiability and integration; could be associated to their difficulties with the limit concept; (d) a high percentage of students have a static view of mathematics (Cornu 1991; Sierpiñska 1987) which forces them to only deal with a very specific calculation placed before them and such students are likely to have difficulties with the concept of a limit of a function; (e) the symbol $\lim_{x \to a} f(x)$ is an example of a *procept* since it represents both the process of getting to a specific value and the value of the limit of the function itself; (f) there is no universal algorithm that works in all cases to find the value of a limit of a function; (g) since the limit of a function is often something that could actually never be attained this contributes to the difficulty students have in constructing a process conception of a limit of a function (Cottrill et al. 1996; Dubinsky 2010);

and (h) the students' difficulty to understand this concept is possibly the result of many of them not having appropriate mental structures at the process, object and schema levels (in the context of APOS (action-process-object-schema) Theory). The paper by Brijlall and Maharaj (2013) suggests that a major obstacle could arise if one did not adequately reorganise his or her schemata, for evaluating limits of a function with that of continuity of the function at x=α. In such a case the two schemata could be operating in compartments. It follows that each of those observations could impact on students' understanding of continuity of a function.

The above guided the conceptual framework and methodology for this paper.

Conceptual Framework

The conceptual framework for outlining possible in-course diagnostics on elementary logic, and the limit and continuity of a function was guided by the literature review and the following principles:

1. There is a conceptual hierarchy in the body of mathematics.
2. To study mathematics students should have good work habits and they should know what is meant by these. [Focused on in the paper by Maharaj and Wagh (2014).]
3. It is important for the learning outcomes for the unit or module to be clearly documented (Council of Regional Accrediting Commissions 2004). Further, students should know explicitly at the outset the learning outcomes expected of them.
4. For effective learning to occur it is not good enough for an instructor (teacher, lecturer or tutor) to be aware of the technical knowledge outcomes of a course/module. The documented learning outcomes should guide the formulation of suitable diagnostic questions, for students.
5. When students attempt the diagnostic questions there should be provisions for remedial activity, to overcome their identified shortcomings.

METHODOLOGY

This was informed by the literature review, conceptual framework and study of the aims and content for differential calculus (Math130) mod-

ule. The aim and content as indicated in the handbook of the Faculty of Science and Agriculture (2010); which is in the public domain; was looked at. These are indicated below, the parts indicated in italics is the researcher's emphasis:

Aim

To introduce and develop the Differential Calculus as well as the fundamentals of proof technique and rudimentary logic.

Content

Fundamental Concepts - *elementary logic*, proof techniques. Differential Calculus - Functions, graphs and inverse functions, *limits and continuity*, the derivative, techniques of differentiation, applications of derivatives, antiderivatives.

Then the researcher used his experience relating to teaching at both secondary and tertiary education institutions to formulate and document:

1. In-course expected learning outcomes for the sections on *elementary logic*, the *concept of a limit a function* and *continuity of a function*. These were formulated for the Math130 module by studying its aim and content (as indicated above), and also past assessment questions. [The focus of this paper.]
2. In-course diagnostics for elementary logic, and the limit and continuity of a function. The identified learning outcomes were used to formulate questions on course content for elementary logic and the two concepts, limit of a function and continuity of a function.

FINDINGS AND DISCUSSION

These are presented in the following order: (a) elementary logic, (b) limit of a function, and (c) continuity of a function.

Elementary Logic

The learning outcomes for elementary logic are followed by the diagnostic questions that were formulated.

Learning Outcomes

We expect students to be able to:

1.1. identify the 'if' and 'then' part of a statement

2. construct the negation of a statement
   - ➢ Recognize that the *negation* of 'for all values of the variable the condition is true' is 'there exists at least one case for which the condition is false'
   - ➢ Recognize that the *negation* of 'for some value of the variable the condition is true' is 'for every value of the variable the condition is false'

3. recognize when an implication is false
   - ➢ Recognize that 'an implication is false when the if condition is true and the then condition is false'

4. recognize when an implication is true
   - ➢ Recognize that 'an implication is true when the then condition is false forces the if condition to be false'. An implication is true in the following three case

p	q	$p \rightarrow q$
T	T	T
F	F	T
F	T	T

5. connect two statements with an 'and'
   - ➢ Recognize that 'both statements must be satisfied'

6. connect two statements with 'or'
   - ➢ Recognize that 'one of the two statements must be satisfied'

7. identify equivalent statements
   - ➢ Recognize that they are *either* 'both true' or 'both false'

Diagnostic Questions

The in-course diagnostics for elementary logic (see Table 1) concerns the working knowledge on negation, implication and equivalent statements with special reference to the ability to follow mathematical arguments.

An examination of the diagnostic questions for elementary logic (see Table 1) should indicate that: These were guided by the formulation of expected learning outcomes (Adams 2006). The researcher concentrated on the principles of correct reasoning, the languages and rules of reasoning which Lau and Chan (2013a) noted are precisely and carefully defined, for example see numbers 2 and 8. There is also a focus of how one statement is related to another (Lau and Chan 2013b), for example see number 4. Further the characteristics of a good argument (Lau

Table 1: Diagnostic questions for elementary logic

S.No.	Question	Answer
1.	Identify the 'if' and 'then' part of the following statement. The denominator of a rational number x is always a non-zero integer.	If part: x is a rational number Then part: its denominator is a non-zero integer.
2.	Write the *negation* of the following statement. x is not greater than 4.	There exists a value of x greater than 4; some value of x is greater than 4; x takes a value that is greater than 4.
3.	Write the *negation* of the following statement. Some value of x is a rational number.	Every value of x is a non-rational number; every value of x is irrational.
4.	When is the following statement false? 'If it is Monday then it rains at 4am.'	When it is 4am on Monday and it is not raining.
5.	When is the following statement false? 'If $x < 4$ then $y > 6$.'	When $x < 4$ and $y \leq 6$.
6.	When is the following statement true? 'If the skin is yellow then the lemon is ripe.'	If the lemon is not ripe forces that the skin is not yellow.
7.	When is the following statement true? 'If $x < 4$ then $y > 6$.'	When $y \leq 6$ forces $x \geq 4$.
8.	Write the following expression as two statements connected with the word *and*. $-1 < x < 4$.	$-1 < x$ and $x < 4$; $x > -1$ and $4 > x$
9.	Write the following expression as two statements connected with the word *or*. $x \in (0,1) \cup (2,3)$	$x \in (0,1)$ or $x \in (2,3)$; $0 < x < 1$ or $2 < x < 3$
10.	Write an equivalent statement for: $x \not> 4$	$x \leq 4$
11.	Write an equivalent statement for: $y \neq 6$	$y < 6$ or $y > 6$; $y \in (-\infty, 6) \cup (6, \infty)$
12.	Write an equivalent statement for: $ab = 0$	$a = 0$ or $b = 0$
13.	Write an equivalent statement for: $ab \neq 0$	$a \neq 0$ and $b \neq 0$

and Chan 2013c) at a basic level are also included, for example see numbers 1 and 12. The following sections include diagnostic questions which focus on a good argument at a higher level. This was to cater for the views of those colleagues who believe that elementary logic should be integrated in the teaching of relevant content for different sections and also in the writing out of solutions to problems, as remarked on in the literature review. The researcher's discussions with some interested researchers at a report back meeting during the 2013 HP Global Catalyst Education Summit held in Sao Paulo (Brazil) indicated that the majority of them were of the opinion that elementary logic should be integrated in the teaching of the different sections.

Limit of a Function

The expected learning outcomes for limit of a function and the possible diagnostic questions to attain these are indicated.

Learning Outcomes

We expect students to be able to:
1. recognize the mathematical representation and meaning of the concept 'approaches'
2. recognize the limit of a given sequence of numbers if it exists
3. recognize the mathematical representation and meaning of the concept 'one sided limits'
4. determine one sided limits of split functions
5. determine the limit of standard functions when the variable approaches a given value, if the limit exists
6. compute and recall standard limits
7. deduce and recall laws of limits and operations on limits
8. compute required limits using applications of the standard limits

Diagnostic Questions

This concerns the meaning of a limit and its mathematical representation, one-sided limits, computation of one-sided limits, computation of standard limits, using the laws of limits to compute non-standard limits.

An examination of diagnostic questions on the concept of a limit of a function (see Table 2)

indicates: The expected learning outcomes guided the formulation of the diagnostic questions (Adams 2006). Aspects of elementary logic were incorporated into those questions, for example see numbers 3, 5 and 14. With regard to the literature review (Maharaj 2010) questions were also phrased questions to focus on the relatively higher mental structures at the process, object and schema levels (for example see numbers 2, 8 and 15 respectively). Further informed by the literature review questions were formulated that use different techniques to evaluate the limit of different functions; for example see numbers 6, 9 to 13 and 16. This was to expose students to the idea that there is no universal algorithm that works in all cases to find the value of a limit of a function (as noted in the literature review). The thinking here was that this could possibly help those students who had a static view of mathematics (Cornu1991; Sierpiñska 1987); which forces them to only deal with very specific calculations placed before them; to develop flexibility (Maharaj 2010, 2013) in their thinking.

Continuity of a Function

The expected learning outcomes for continuity of a function and possible diagnostic questions to attain these are indicated.

Learning Outcomes

We expect a student to be able to:
1. recall the defining conditions for continuity of a function at a point
2. recall the defining conditions for continuity of a function on a given domain
3. determine whether a given function is continuous at a point or on a given domain
4. recall the domain for continuity of standard functions
5. determine and recall the domain of continuity of various new functions, on the basis of knowledge of continuity of standard functions
6. recognize the standard observations emerging from the continuity of a function

Diagnostic Questions

This concerns the continuity of functions at a point in a given domain or on the domain,

Table 2: Diagnostic questions for limit of a function

S.No.	Question	Answer
1.	Interpret the following and state it in words. $$x \to 2, f(x) \to c$$	As x approaches 2, $f(x)$ approaches c.
2.	The limit of the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is ___	0
3.	True or false? If $\lim_{x \to a} f(x) = c$ then c is always in the range of the function f.	False. For example $\lim_{x \to \infty} \frac{1}{x} = 0$, but 0 is not in the range of the function $f(x) = \frac{1}{x}$.
4.	Let f be a function defined on an open interval containing a, except possibly at a itself. Use the precise definition of the limit to indicate what is meant by: $$\lim_{x \to a} f(x) = L.$$	For every $\epsilon > 0$ there exists $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$.
5.	Consider the function $$f(x) = \begin{cases} 2^{-x}, & -\infty < x \le 0 \\ x^2, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$$ Compute the following: a) $\lim_{x \to 0^-} f(x)$ b) $\lim_{x \to 0^+} f(x)$ c) $\lim_{x \to 1^-} f(x)$ d) $\lim_{x \to 1^+} f(x)$ Does the following limit exist? Explain. $$\lim_{x \to 1} f(x)$$	a) 1 b) 0 c) 1 d) 0 $\lim_{x \to 1} f(x)$ does not exist since $$\lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$$
6.	Compute the infinite limit: a) $\lim_{x \to \pi^-} \operatorname{cosec} x$ b) $\lim_{x \to \pi^+} \operatorname{cosec} x$	a) ∞ b) $-\infty$
7.	Compute: $\lim_{x \to 2} \dfrac{x - 2}{x^2 - 4}$	$\dfrac{1}{4}$
8.	Compute: $$\lim_{x \to 36} \dfrac{\sqrt{x} - 6}{x - 36}$$	$\dfrac{1}{12}$
9.	Compute: $\lim_{x \to 2} \dfrac{x - 2}{x^3 - 8}$	$\dfrac{1}{12}$
10.	Compute: $\lim_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$	1
11.	Compute: $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|}$	-1
12.	Compute: $\lim_{x \to 3} \dfrac{\sqrt{x + 1} - 2}{x - 3}$	$\dfrac{1}{4}$

Table 2: Contd...

S.No.	Question	Answer
13.	Compute: $$\lim_{x\to\infty}\frac{-3x^2+3x-8}{-6x^2+10}$$	$\frac{1}{2}$
14.	State the conditions and the conclusion of the Sandwich Theorem (which is also known as the Squeeze Theorem).	Conditions: On an open interval which includes $x=a$, $f(x)\le g(x)\le h(x)$, and $\lim_{x\to a}f(x)=k$ and $\lim_{x\to a}h(x)=k$. Conclusion: $\lim_{x\to a}g(x)=k$
15.	Use the Sandwich Theorem to compute: $\lim_{x\to0}\left[x^2\cos\left(\frac{1}{x^2}\right)\right]$	0
16.	If $\lim_{x\to0}\frac{f(x)}{x^2}=4$, find $\lim_{x\to0}f(x)$.	0

and applications of these ideas to standard functions and non-standard functions.

The diagnostic questions for continuity of functions (see Table 3) indicate that: They were informed by the documentation of the relevant expected learning outcomes (Adams 2006) for continuity of a function concept. As indicated in the discussion for the section on elementary logic, questions were formulated to gradually extend and test the ability of students to provide good arguments. The researcher believes that this is so since many of these questions focus on the characteristics of a good argument as noted by Lau and Chan (2013c), in the literature review. For example see the diagnostic questions 4, 6 and 8. Further, note that in the context of APOS Theory some of the questions test for attainment by students of higher order mental structures, which Maharaj (2010) found was lacking in many students in his paper. For example questions 4, 6 and 8 also focus on conceptualising the different objects and modifying existing schema to accommodate the type of thinking that is required. This was to also address the issue of schemata for limits and continuity of a function operating in separate compartments (Brijall and Maharaj 2013). The aim here was that exposure to these questions would require students to re-organise and connect their existing schemata.

The learning outcomes and diagnostic questions formulated, for the relevant sections on which this paper focuses, is of interest to people outside South Africa, both in developed and emerging economies. Those sections; elementary logic, and limit and continuity of a function; are important in the study of any differential calculus course offered to students, throughout the world. Informal discussions with students and interested educationists in South Africa, India, Brazil and America suggested that one reason for failure is generally students do not know exactly what is required of them. The formulated learning outcomes should address this aspect, provided they are made available to students. This should be followed by exposing students in good time to suitable diagnostic questions, to help them identify their strengths or weaknesses and to take appropriate remedial measures. Such tests could be paper based or computer based depending on the resources that are available. In addition to the administering of diagnostics tests it is important that some form of feedback on the strengths and weaknesses of a student is planned for and provided, so that if required the student could take appropriate remedial measures. The researcher's opinion is that such diagnostic tests for particular sections in calculus should lead to an improvement of student performance, as was observed by some

Table 3: Diagnostic questions for continuity of a function

S.No.	Question	Answer
1.	State the conditions for a function $f(x)$ to be continuous at a point where $x = a$.	a) $f(a)$ exists b) $\lim_{x \to a} f(x)$ exists c) $\lim_{x \to a} f(x) = f(a)$
2.	When is a function $f(x)$ continuous on a given domain?	When it is continuous at every value in the domain.
3.	Consider the function: $$h(x) = \begin{cases} \frac{1}{x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$ Discuss the continuity of this function on its domain $(-\infty, \infty)$.	The function is discontinuous at $x = 0$ and continuous at all other values on its domain.
4.	Consider the function: $$g(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$$ Discuss the continuity of this function on its domain.	The domain of g is $(-\infty, \infty)$. a) Since $y = x^2 - 1$ is a polynomial function it is continuous everywhere and in particular on $(-\infty, 1)$. b) The square root function $y = \sqrt{x-1}$ is continuous on its domain $[1, \infty)$. c) Discussion of continuity at $x = 1$: $$g(1) = \sqrt{1-1} = 0$$ $$\lim_{x \to 1^-}(x^2 - 1) = 0$$ $$\lim_{x \to 1^+} \sqrt{x-1} = 0$$ $$\therefore \lim_{x \to 1} g(x) = 0 = g(1)$$ From which it follows that $g(x)$ is continuous at $x = 1$. Conclusion: From the above it follows that the function $g(x)$ is continuous on its domain.
5.	For a function $f(x)$ that is continuous at $x = a$, what is the easy way of evaluating $\lim_{x \to a^+} f(x)$.	$f(x)$ is continuous at $x = a$ $\Rightarrow \lim_{x \to a} f(x) = f(a)$ $\Rightarrow \lim_{x \to a} f(x) = f(a)$
6.	Use continuity to discuss the evaluation of the following limit: $$\lim_{x \to \frac{\pi}{2}} \ln(\sin x)$$	The function $y = \ln(\sin x)$ is continuous provided $\sin x > 0$. Observe that $\sin x > 0$ in the open interval $(0, \pi)$ which contains $x = \frac{\pi}{2}$. From this we get: $$\lim_{x \to \frac{\pi}{2}} \ln(\sin x)$$ $$= \ln\left(\sin\frac{\pi}{2}\right) = \quad 0$$
7.	State the Intermediate Value Theorem	If $f(x)$ is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$ then for any c between $f(a)$ and $f(b)$ there exists x_0 in the open interval (a, b) such that $c = f(x_0)$.
8.	Show that the equation $x^5 + 2x = x^2 - 3$ has at least one real root in the open interval $(-1, 0)$	Let $h(x) = x^5 + 2x - x^2 + 3$. Since this is a polynomial function it

Table 3: Contd...

S.No.	Question	Answer
		is continuous on the closed interval $[-1,0]$. $h(-1) = -1$ and $h(0) = 3$. By the Intermediate Value Theorem there exists some d in $(-1,0)$ such that $h(d) = 0$.

researchers (Betts et al. 2011) who studied the impact of diagnostic tests mainly based on pre-course content. In the context of large student numbers and where computers are available, well planned and administered diagnostic tests with suitable student feedback should have the following benefits: (1) students would get prompt feedback on their strengths and weaknesses well before grading tests or examinations, (2) students could develop a culture where they take more responsibility for their own learning, and (3) this should free staff members from attending to a large number of student queries based on basic course-work.

In the researcher's opinion, the outline of in-course diagnostics for elementary logic and limits and continuity of a function provides a good platform for research on diagnostic testing relating to those sections, in both the developed and developing economies. The reader is encouraged to implement the material produced in this paper, even in a modified form to suite his/ her needs and according to the availability of resources. This researcher would be interested in the success or lack thereof of such implementations and any constructive feedback on the use of learning outcomes and diagnostics with regard to the relevant in-course content.

CONCLUSION

The researcher was able to identify and document clearly the expected learning outcomes with regard to the sections on elementary logic and the two concepts, limit and continuity of a function. These enabled the development of in-course diagnostics on elementary logic and on the two concepts, the limit and continuity of a function, for differential calculus as offered at the UKZN (in particular). The formulation of diagnostic questions for the sections on limit of a function and continuity of a function took into consideration relevant aspects pertaining to these sections and also the section on elementary logic, as informed by the literature review. Those questions also tried to accommodate the two divergent views prevalent at UKZN on how elementary logic should be taught.

RECOMMENDATIONS

It is the researcher's view that the diagnostic questions given in Tables 1 to 3 are a good representation of samples questions that could be used to check whether students have attained the expected learning outcomes that were documented.Since much time and effort were spent to document the relevant expected learning outcomes and sample diagnostic questions,both of these should be used by the relevant stakeholders (lecturers, tutors or mentors) of first year undergraduate mathematics' students. Such use could benefit students at UKZN and at institutions in developed or developing countries. Those interested could investigate whether the documented expected learning outcomes and sample diagnostic questions do what they are intended for. It is recommended that other studies should be planned by the relevant stakeholders for the implementation and reporting of the effect of such diagnostic questions on their students' understanding of the relevant sections.

REFERENCES

Adam S 2006. An Introduction to Learning Outcomes. From <http://books.google.co.in/books?id= mxN-FkhZX UisCandprintsec= frontcover andsource= gbs_ge _summary_ randcad= 0#v= onepageandqandf =false> (Retrieved on 7 February 2013).

Betts JR, Hahn Y, Zau AC 2011. Does Diagnostic Math Testing Improve Student Learning? From <http://mdtp.ucsd.edu/pdf/PPIC_MDTP_Student Learning. pdf.> (Retrieved on 7 February 2013).

Cornu B 1991. Limits. In: D Tall (Ed.): *Advanced Mathematical Thinking*. Dordrecht: Kluwer Academic Publishers, pp. 153-166

Brijlall D, Maharaj A 2013. Pre-service teachers understanding of continuity of functions in the context of assessment items. *South African Journal of Higher Education*, 27(4): 797-814.

Cottrill J, Nichols D, Schwingendorf K, Thomas K, Vidakovic D 1996. Understanding the Limit Concept: Beginning with a Coordinated Process Schema. Journal of Mathematical Behavior, 15(2): 167-192. From<http://homepages.ohiodominican.edu/ ~cottrilj/concept-limit.pdf> (Retrieved on 19 July 2013).

Council of Regional Accrediting Commissions 2004. Regional Accreditation and Student Learning: A Guide for Institutions and Evaluators. Atlanta: Southern Association of Colleges and Schools. From <http:// www.sacscoc.org/pdf/handbooks/Guide For Institutions. PDF> (Retrieved on 7 February 2013).

Davis RB, Vinner S 1986. The notion of limit: Some seemingly unavoidable misconception stages. *Journal of Mathematical Behavior,* 5: 281-303.

Dubinsky E 2010. The APOS Theory of Learning Mathematics: Pedagogical Applications and Results. *Paper presented at the Eighteenth Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education*, 18-21 January 2010, Durban, South Africa.

Lau J, Chan J 2013a. What is Logic?From <http:// philosophy. hku.hk/think/logic/whatislogic. php> (Retrieved on 18 February 2013).

Lau J, Chan J 2013b. Statements. From <http:// philosophy.hku.hk/think/logic/statements.php> (Retrieved on 18 February 2013).

Lau J, Chan J 2013c. Good Arguments. From <http:// philosophy.hku.hk/think/arg/goodarg.php> (Retrieved on 19 February 2013).

Learning and Technology Support Network Maths Team Project 2003. Diagnostic Testing for Mathematics. From <http://www.mathstore.ac.uk/head-ocs/diagnostic_test.pdf> (Retrieved on 7 March 2013).

Li L, Tall D 1993. Constructing Different Concept Images of Sequences and Limits by Programming. In: I Hirabayashi, N Nohda, K Shigematsu, F Lin (Eds.): Proceedings of the 17th Conference of theInternational Group for the Psychology of Mathematics Education 2, pp. 41-48. From <http:// www.warwick.ac.uk/staff/David.Tall/pdfs/dot1993e-lan-li-pme.pdf> (Retrieved on 19 July 2013).

Maharaj A 2010. An APOS analysis of students' understanding of the concept of a limit of a function. *Pythagoras*, 71: 41-52.

Maharaj A 2013. An APOS analysis of natural science students' understanding of derivatives. *South African Journal of Education*, 33(1): 146-164.

Maharaj A, Wagh V 2014. An outline of possible precourse diagnostics for differential calculus. *The South African Journal of Science*, 110(7/8): 27-33.

Maharaj A(pre-print). An outline of possible in-course diagnostics for functions. *International Journal of Educational Sciences,* In Press.

Maharajh N, Brijlall D, Govender N 2008. Pre-service mathematics students' notions of the conceptdefinition of continuity in calculus through collaborative instructional design worksheets. *African Journalof Research in Mathematics, Science and Technology Education,* 12: 93-108.

Monaghan J, Sun S, Tall D 1994. Construction of the Limit Concept with a Computer Algebra System. In: JP Ponte and JF Matos (Eds.): Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education (3: 279-286). Lisbon: University of Lisbon. From <http:// www.warwick.ac.uk/staff/David.Tall/pdfs/ dot1994c-monhn-sun-pme.pdf> (Retrieved on 19 July 2013).

Sierpiñska A 1987. Humanities students and epistemological obstacles related to limits. *Educational Studies in Mathematics,* 18: 371-397.

Tall D 1992. The transition to advanced mathematical thinking: Functions, limits, infinity and proof. In: DA Grouws (Ed.): *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan Publishing, pp. 495-511.

Tall D, Vinner S 1981. Concept image and concept definition in mathematics with particular reference

to limits and continuity. *Educational Studies in Mathematics,* 12: 151-169.

The California State University 2012. Mathematics Diagnostic Testing Project. From <http://mdtp. ucsd.edu/OnLineTests.shtml> (Retrieved on 2 February 2013).

Williams SR 1991. Models of limit held by college calculus students. *Journal for Research in Mathematics Education,* 22: 219-236.